Opener

$\int \sec^2 x\,dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\dfrac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

$\int \dfrac{5}{1+x^2}\,dx =$

(A) $\dfrac{-10x}{\left(1+x^2\right)^2} + C$ (B) $\dfrac{5}{2x}\ln\left(1+x^2\right) + C$ (C) $5x - \dfrac{5}{x} + C$

(D) $5\arctan x + C$ (E) $5\ln\left(1+x^2\right) + C$

Which of the following is equal to $\int \dfrac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin\dfrac{x}{5} + C$ (B) $\cancel{\arcsin x + C}$ (C) $\dfrac{1}{5}\arcsin\dfrac{x}{5} + C$

(D) $\cancel{\sqrt{25-x^2} + C}$ (E) $\cancel{2\sqrt{25-x^2} + C}$

6-1 day 1 Differential Equations

Learning Objectives:

I can Solve a Differential Equation

A differential equation is an equation involving a derivative like:

$$\frac{dy}{dx} = 3xy^2$$

Dec 6-11:18 AM

Ex1. Solve the differential equation

(x, y)

1.) $\frac{dy}{dx} = 3xy^2$ given the initial condition $\boxed{(2,1)}$

$$\left[\frac{dy}{dx} = 3xy^2\right] \cdot dx$$

$$\frac{dy}{y^2} = \frac{3x\cancel{y^2}dx}{\cancel{y^2}}$$

$$\int \frac{1}{y^2}\,dy = \int 3x\,dx$$

$$\int y^{-2}\,dy = \int 3x\,dx$$

$$-y^{-1} = \frac{3}{2}x^2 + C$$

$$y \cdot -\frac{1}{y} = \frac{3}{2}x^2 + C \cdot y$$

$$-1 = \left(\frac{3}{2}x^2 + C\right) \cdot y$$

$$\frac{-1}{\frac{3}{2}x^2 + C} = y$$

general solution to the diffyQ

i.c. $(2,1)$

$$\frac{-1}{\frac{3}{2}(2)^2 + C} = 1$$

$$\frac{-1}{\frac{3}{2}\cdot 4 + C} = 1$$

$$\frac{-1}{6 + C} = 1$$

$$-1 = 6 + C$$

$$-7 = C$$

$$\frac{-1}{\frac{3}{2}x^2 - 7} = y$$

Specific solution to the diffy Q

Dec 6-11:18 AM

Steps to Solving a DiffyQ

1.) Separate the variables

2.) Integrate both sides

3.) Solve for y (if possible)

4.) Using initial condition to find C

you can reverse the order for steps 3 & 4. This is often easier.

Dec 6-11:18 AM

2.) $\frac{dy}{dx} = \frac{3x^2}{2y}$ given the initial condition (2,5)

1. $dy(2y) = dx(3x^2)$
2. $\int 2y(dy) = \int 3x^2(dx)$

$y^2 = x^3 + C$

3. $y = \sqrt{x^3 + C}$
4. $5 = \sqrt{8 + C}$

$25 = 8 + C$

$C = 17.$

$\boxed{y = \sqrt{x^3 + 17}}$

Dec 6-11:18 AM

3.) $\frac{dy}{dx} = y\sqrt{x}$ given the initial condition

$(4,-e^2)$

$dy = y\sqrt{x}\,dx \qquad \frac{1}{y}dy = \sqrt{x}\,dx$

$\int \frac{1}{y}dy = \int \sqrt{x}\,dx \qquad \ln|y| = \frac{2}{3}x^{3/2} + C$

$\ln|-e^2| = \frac{2}{3}(4)^{3/2} + c$

$\ln e^2 = \frac{2}{3}(8) + c$

$\frac{6}{3} \quad 2 = \frac{16}{3} + c$

$-\frac{16}{3} \quad -\frac{16}{3}$

$-\frac{10}{3} = c$

$\cancel{\ln}|y| = \frac{2}{3}x^{3/2} - \frac{10}{3}$

$e \qquad e^{2/3x^{3/2} - 10/3}$

$|y| = e^{2/3x^{3/2} - 10/3}$

$y = \pm e$

i.c. $(4,-e^2)$

$$\boxed{y = -e^{2/3x^{3/2} - 10/3}}$$

Ex2. Find the general solution to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\int y\,dy = \int -x\,dx$$

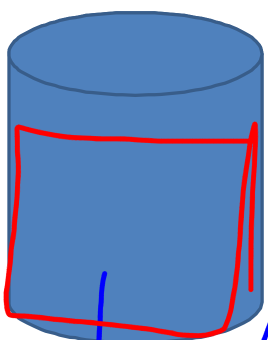
$$\left[\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C\right]\cdot 2$$

$$y^2 = -x^2 + C$$

$$y = \pm\sqrt{-x^2 + C}$$

Toricelli's Law

If you drain a tank from the bottom, the rate at which the liquid runs out is a constant times the square root of the depth of the liquid.



$$r = k\sqrt{d}$$

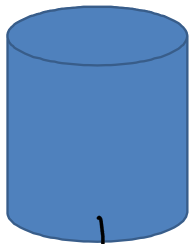
$\frac{dV}{dt}$

r = rate at which liquid drains

k=constant

d=depth of the liquid

Ex3. A right cylindrical tank with a radius of 5 ft and a height of 16 ft that was initially full is being drained at the rate of $.5\sqrt{h}$ ft³/min.



a.) Find a formula for the depth of the water in the tank at time t.

$$V = \pi r^2 h$$

$$V = 25\pi h$$

$$\frac{dv}{dt} = 25\pi \frac{dh}{dt}$$

$$\frac{dv}{dt} = -.5\sqrt{h}$$

$$dt\left[-\frac{1}{2}\sqrt{h}\right] = \left[25\pi \frac{dh}{dt}\right]dt$$

$$\frac{-\frac{1}{2}\sqrt{h}\,dt}{h^{1/2}} = \frac{25\pi\,dh}{h^{1/2}}$$

$$-\frac{1}{2}\,dt = \frac{25\pi}{h^{1/2}}\,dh$$

$$\int -\frac{1}{2}\,dt = \int 25\pi h^{-1/2}\,dh$$

$$-\frac{1}{2}t = 50\pi h^{1/2} + C$$

$$\frac{-50\pi h^{1/2}}{-50\pi} = \frac{\frac{1}{2}t + C}{-50\pi}$$

$$\left(h^{1/2}\right)^2 = \left(\frac{\frac{1}{2}t + C}{-50\pi}\right)^2$$

$t = 0$
$h = 16$

$$h = \left(\frac{\frac{1}{2}t + C}{-50\pi}\right)^2 \qquad \sqrt{16} = \sqrt{\left(\frac{C}{-50\pi}\right)^2}$$

$$-50\pi(4) = \left(\frac{C}{-50\pi}\right)\cdot -50\pi$$

$$-200\pi = C$$

$$h = \left(\frac{\frac{1}{2}t - 200\pi}{-50\pi}\right)^2$$

$$\boxed{h = \left(-\frac{1}{100\pi}t + 4\right)^2}$$

b.) How long does it take to completely drain the tank?

$$h = \left(-\frac{1}{100\pi}t + 4\right)^2$$

$$\sqrt{0} = \sqrt{\left(-\frac{1}{100\pi}t + 4\right)^2}$$

$$0 = -\frac{1}{100\pi}t + 4$$

$$\left(-4 = -\frac{1}{100\pi}t\right) \cdot -100\pi$$

$$\boxed{400\pi \text{ min} = t}$$

Dec 9-11:48 AM

Homework

pg 327 #DiffyQ ws, 59, 60, 63

Dec 6-11:18 AM